

Challenges of an accelerating universe in string theory

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and

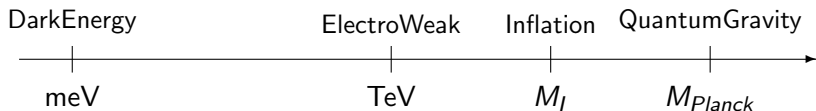
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Universe evolution: based on positive cosmological constant

- Dark Energy
simplest case: infinitesimal (tuneable) +ve cosmological constant
- Inflation (approximate de Sitter)
describe possible accelerated expanding phase of our universe



The cosmological constant in Supergravity

Highly constrained: $\Lambda \geq -3m_{3/2}^2$

- equality \Rightarrow AdS (Anti de Sitter) supergravity

$m_{3/2} = W_0$: constant superpotential

- inequality: dynamically by minimising the scalar potential

\Rightarrow uplifting Λ and breaking supersymmetry

- Λ is not an independent parameter for arbitrary breaking scale $m_{3/2}$

What about breaking SUSY with a $\langle D \rangle$ triggered by a constant FI-term?

standard supergravity: possible only for a gauged $U(1)_R$ symmetry:

absence of matter $\Rightarrow W_0 = 0 \rightarrow$ dS vacuum Friedman '77

- exception: non-linear supersymmetry [8]

$$K = X\bar{X} \quad ; \quad W = fX + W_0$$

$X \equiv X_{NL}$ nilpotent goldstino superfield [6]

$$X_{NL}^2 = 0 \Rightarrow X_{NL}(y) = \frac{\chi^2}{2F} + \sqrt{2}\theta\chi + \theta^2 F$$

$$\Rightarrow V = |f|^2 - 3|W_0|^2 \quad ; \quad m_{3/2}^2 = |W_0|^2$$

- V can have any sign **contrary to global NL SUSY**
- NL SUSY in flat space $\Rightarrow f = \sqrt{3} m_{3/2} M_p$
- R-symmetry is broken by W_0

gauge invariant at the Lagrangian level but non-local

becomes local and very simple in the unitary gauge

Global supersymmetry:

$$\mathcal{L}_{\text{FI}}^{\text{new}} = \xi_1 \int d^4\theta \frac{W^2 \bar{W}^2}{\mathcal{D}^2 W^2 \bar{\mathcal{D}}^2 \bar{W}^2} D\overset{\text{gauge field-strength superfield}}{W} = -\xi_1 D + \text{fermions}$$

It makes sense only when $\langle D \rangle \neq 0 \Rightarrow$ SUSY broken by a D-term

Supergravity generalisation: straightforward

unitary gauge: goldstino = $U(1)$ gaugino = 0 \Rightarrow standard sugra $-\xi_1 D$

New FI term in supergravity

Pure sugra + one vector multiplet \Rightarrow [4]

$$\mathcal{L} = R + \bar{\psi}_\mu \sigma^{\mu\nu\rho} D_\rho \psi_\nu + m_{3/2} \bar{\psi}_\mu \sigma^{\mu\nu} \psi_\nu - \frac{1}{4} F_{\mu\nu}^2 - \left(-3m_{3/2}^2 + \frac{1}{2} \xi_1^2 \right)$$

- $\xi_1 = 0 \Rightarrow$ AdS supergravity
- $\xi_1 \neq 0$ uplifts the vacuum energy and breaks SUSY
e.g. $\xi_1 = \sqrt{6} m_{3/2} \Rightarrow$ massive gravitino in flat space

New FI-term introduces a cosmological constant in the absence of matter

Presence of matter \Rightarrow non trivial scalar potential net result: $\xi_1 \rightarrow \xi_1 e^{K/3}$
but breaks Kähler invariance

However new FI-term in the presence of matter is not unique

Question: can one modify it to respect Kähler invariance?

Answer: yes, constant FI-term + fermions as in the absence of matter

\Rightarrow constant uplift of the potential I.A.-Chatrabhuti-Isono-Knoops '18
Jang-Porrati '21

In general $\xi_1 \rightarrow \xi_1 f(m_{3/2}[\phi, \bar{\phi}])$ I.A.-Rondeau '99

It can also be written in $N = 2$ supergravity

I.A.-Derendinger-Farakos-Tartaglino Mazzucchelli '19

Swampland de Sitter conjecture

String theory: vacuum energy and inflation models
related to the moduli stabilisation problem

Difficulties to find dS vacua led to a conjecture:

$$\frac{|\nabla V|}{V} \geq c \quad \text{or} \quad \min(\nabla_i \nabla_j V) \leq -c' \quad \text{in Planck units}$$

with c, c' positive order 1 constants

Ooguri-Palti-Shiu-Vafa '18

Dark energy: forbid dS minima but allow maxima

Inflation: forbid standard slow-roll conditions

Assumptions: heuristic arguments, no quantum corrections

→ here: explicit counter example

Moduli stabilisation in type IIB

Compactification on a Calabi-Yau manifold $\Rightarrow N = 2$ SUSY in 4 dims

Moduli: Complex structure in vector multiplets ($N = 1$ vector + chiral)

Kähler class & dilaton in hypermultiplets (two $N = 1$ chiral)

\Rightarrow decoupled kinetic terms

turn on appropriate 3-form fluxes (primitive self-dual) $\Rightarrow N = 1$ SUSY
field-strengths of 2-index antisymmetric gauge potentials

+ orientifolds and D3/D7-branes

vectors and RR companions of geometric moduli are projected away \Rightarrow

all moduli in $N = 1$ chiral multiplets + superpotential for the

complex structure & dilaton \rightarrow fixed in a SUSY way Frey-Polchinski '02

Kähler moduli: no scale structure, vanishing potential (classical level)

Moduli stabilisation in type IIB

flux generated superpotential:

$$W = \int_{\text{CY}} G_3 \wedge \Omega \quad ; \quad G_3 = F_3^{\text{RR}} - \Phi H_3^{\text{NS}} \quad , \quad \Phi = C_0^{\text{RR}} + ie^{-\phi}$$

holomorphic 3-form

⇒ 3-brane charge:

$$Q_3 = - \int_{\text{CY}} H_3 \wedge F_3 > 0 \quad \Rightarrow \quad \text{need orientifold planes } O_3 \text{ and branes}$$

⇒ leftover Kähler moduli effective supergravity: [14]

$$K = -2 \ln \mathcal{V} \quad ; \quad W = W_0 \quad \Rightarrow \quad \text{vanishing scalar potential}$$

Stabilisation of Kähler moduli

Non perturbative superpotential from gaugino condensation on D-branes

⇒ stabilisation in an AdS vacuum Derendinger-Ibanez-Nilles '85

Uplifting using anti-D3 branes Kachru-Kalosh-Linde-Trivedi '03

or D-terms and perturbative string corrections to the Kähler potential

Large Volume Scenario (LVS) Conlon-Quevedo et al '05

Ongoing debate on the validity of these ingredients in full string theory

While perturbative stabilisation has the old Dine-Seiberg problem

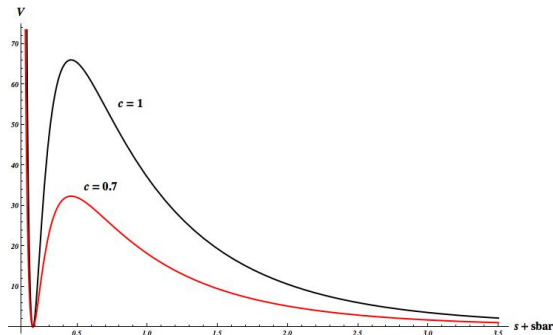
put together 2 orders of perturbation theory violating the expansion

possible exception known from field theory:

logarithmic corrections → Coleman-Weinberg mechanism [13]

The Dine-Seiberg problem

Runaway potential towards vanishing string coupling or large volume



⇒ if there is perturbative minimum, it is likely to be at strong coupling
or string size volume

Analogy with Coleman-Weinberg symmetry breaking

Effective potential in massless $\lambda\Phi^4$

$$V = \left\{ \sum_{N>1} c_N \lambda^N(\Phi) \right\} \Phi^4 \Rightarrow \text{minimum at } \lambda = 0 \text{ or } \mathcal{O}(1)$$

C-W perturbative symmetry breaking needs 2 couplings + logs: [18]

$$V_{\text{C-W}} = \left(\lambda + c_1 e^4 \ln \frac{|\Phi|^2}{\mu^2} \right) |\Phi|^4 \Rightarrow |\Phi|_{\text{min}}^2 \propto \mu^2 e^{-\frac{\lambda}{c_1 e^4}}$$

both λ and e are weak < 1

realising this proposal in string theory:

- replace gaugino condensation by log corrections in the F-part potential
- use D-term uplifting as in LVS

Log corrections in string theory:

localised couplings + closed string propagation in $d \leq 2$

Effective propagation of massless bulk states in $d \leq 2 \Rightarrow$ IR divergences [18]

$d = 1$: linear, $d = 2$: logarithmic

\Rightarrow corrections to (brane) localised couplings

depending on the size of the bulk due to local closed string tadpoles

I.A.-Bachas '98

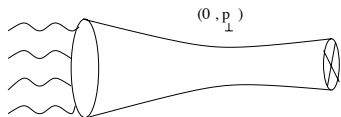
e.g. threshold corrections to 4d gauge coupling

linear dilaton dependence on the 11th dim of M-theory

Type II strings: correction to the Kähler potential \leftrightarrow Planck mass [10] [16]

I.A.-Ferrara-Minasian-Narain '97

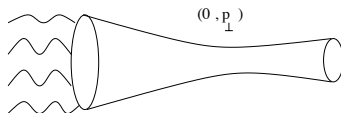
decompactification limit in the presence of branes



(a)

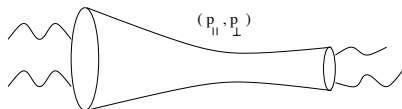
$$\mathcal{A} \sim \frac{1}{V_{\perp}} \sum_{|\vec{p}_{\perp}| < M_s} \frac{1}{p_{\perp}^2} F(\vec{p}_{\perp})$$

$$V_{\perp} = R^d \quad \vec{p}_{\perp} = \vec{n}/R$$



(b)

$$R \gg l_s \Rightarrow$$



(c)

$$\mathcal{A} \sim \begin{cases} \mathcal{O}(R) & \text{for } d=1 \\ \mathcal{O}(\log R) & \text{for } d=2 \\ \text{finite} & \text{for } d > 2 \end{cases}$$

$$\text{local tadpoles: } F(\vec{p}_{\perp}) \sim \left(2^{5-d} \prod_{i=1}^d (1 + (-)^{n_i}) - 2 \sum_{a=1}^{16} \cos(\vec{p}_{\perp} \cdot \vec{y}_a) \right)$$

Localised gravity kinetic terms

Corrections to the 4d Planck mass in type II strings

Large volume limit: localised Einstein-Hilbert term in the 6d internal space

I.A.-Minasian-Vanhove '02 [18]

10d: $R \wedge R \wedge R \wedge R \rightarrow$ in 4d: $\chi \mathcal{R}_{(4)}$

Euler number = $4(n_H - n_V)$ [21]

$$S_{\text{grav}}^{IIB} = \frac{1}{(2\pi)^7 \alpha'^4} \int_{M_4 \times \mathcal{X}_6} e^{-2\phi} \mathcal{R}_{(10)} + \frac{\chi}{(2\pi)^4 \alpha'} \int_{M_4} \left(2\zeta(3) e^{-2\phi} + \frac{2\pi^2}{3} \right) \mathcal{R}_{(4)}$$

4-loop σ -model \nearrow vanishes for orbifolds

localisation width $w \sim |\chi| l_s = l_p^{(4)}$

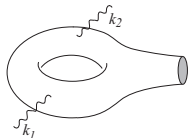
in agreement with general arguments of localised gravity

Dvali-Gabadadze-Porrati '00

perturbative moduli stabilisation I.A.-Chen-Leontaris '18, '19

localised vertices from $\mathcal{R}_{(4)}$ can emit massless closed strings

\Rightarrow local tadpoles in the presence of distinct 7-brane sources

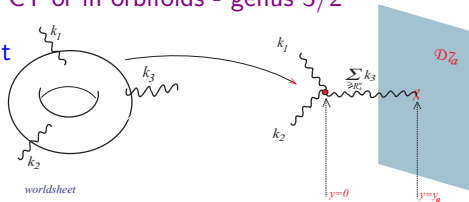


propagation in 2d transverse bulk $\rightarrow \log R_{\perp}$ corrections

exact computation: difficult either in CY or in orbifolds - genus 3/2

computation in the degeneration limit

for Z_N orbifold ($\chi \sim N$)



$$\sim - \sum_{q_{\perp} \neq 0} g_s^2 T N e^{-w^2 q_{\perp}^2 / 2} \frac{1}{q_{\perp}^2 R_{\perp}^2} = -N g_s^2 T \log(R_{\perp} / w) + \dots$$

$T = T_0 / g_s$: brane tension

Kähler potential:

$$\mathcal{K} = -2 \ln \left(\mathcal{V} + \xi + \eta \ln \frac{\mathcal{V}_\perp}{w^2} + \mathcal{O}\left(\frac{1}{\mathcal{V}}\right) \right) = -2 \ln (\mathcal{V} + \eta \ln \mu^2 \mathcal{V}_\perp) \quad [21]$$

$$\xi = -\frac{1}{4} \chi f(g_s); \quad f(g_s) = \begin{cases} \zeta(3) \simeq 1.2 & \text{smooth CY} \\ \frac{\pi^2}{3} g_s^2 & \text{orbifolds} \end{cases} \quad \eta = -\frac{1}{2} g_s T_0 \xi \quad [16]$$

Using 3 mutual orthogonal 7-brane stacks with D-terms (magnetic fluxes) and minimising with respect to transverse volume ratios [13]

$$\Rightarrow V \simeq \frac{3\eta \mathcal{W}_0^2}{\mathcal{V}^3} (\ln \mu^6 \mathcal{V} - 4) + 3 \frac{d}{\mathcal{V}^2} \quad \mathcal{W}_0: \text{ constant superpotential, } d: \text{ D-term}$$

$$\text{dS minimum: } -0.007242 < \frac{d}{\eta \mathcal{W}_0^2 \mu^6} \equiv \rho < -0.006738 \text{ with } \mathcal{V} \simeq e^5 / \mu^6 \quad [20]$$

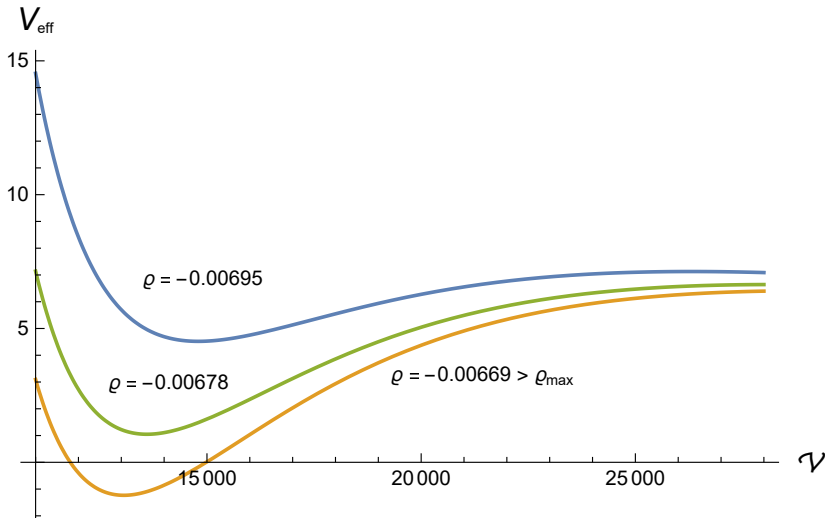
$$V_{D_i} = \frac{d_i}{\tau_i} \left(\frac{\partial K}{\partial \tau_i} \right)^2 = \frac{d_i}{\tau_i^3} + \mathcal{O}(\eta_j)$$

τ_i : world-volume modulus of D7_i-brane stack with $\mathcal{V} = (\tau_1 \tau_2 \tau_3)^{1/2}$

$$\eta_i \equiv \eta \Rightarrow V_{\text{tot}} = \frac{3\eta \mathcal{W}_0^2}{\mathcal{V}^3} (\ln(\mathcal{V} \mu^6) - 4) + \frac{d_1}{\tau_1^3} + \frac{d_2}{\tau_2^3} + \frac{d_3 \tau_1^3 \tau_2^3}{\mathcal{V}^6}$$

minimising with respect to τ_1 and $\tau_2 \Rightarrow \frac{\tau_i}{\tau_j} = \left(\frac{d_j}{d_i} \right)^{1/3} \Rightarrow$

$$V_D = 3 \frac{d}{\mathcal{V}^2} \quad \text{with} \quad d = (d_1 d_2 d_3)^{1/3}$$



2 extrema min+max $\rightarrow -0.007242 < \rho < -0.006738 \leftarrow$ +ve energy [18] [24]

$$\xi = -\frac{1}{4}\chi f(g_s); \quad f(g_s) = \begin{cases} \zeta(3) \simeq 1.2 & \text{smooth CY} \\ \frac{\pi^2}{3} g_s^2 & \text{orbifolds} \end{cases} \quad \eta = -\frac{1}{2} g_s T_0 \xi$$

dS minimum: $-0.007242 < \frac{d}{\eta \mathcal{W}_0^2 \mu^6} \equiv \rho < -0.006738$ with $\mathcal{V} \simeq e^5 / \mu^6$

exponentially large volume:

$$\mu = \frac{e^{\xi/6\eta}}{w} = \sqrt{|\chi|} e^{-\frac{1}{3g_s T_0}} \rightarrow 0 \quad \Rightarrow \quad [18]$$

weak coupling and

large χ or/and \mathcal{W}_0 from 3-form flux to keep ρ fixed

requirement: negative χ ($\eta < 0$) [16] and surplus of D7-branes ($T_0 > 0$)

- Inflaton: canonically normalised $\phi = \sqrt{2/3} \ln \mathcal{V}$ (in Planck units)

- one relevant parameter: ρ or $x = -\ln(-4\rho/3) - 16/3$

$$0 < x < 0.072 \text{ for dS minimum}$$

- extrema $V'(\phi_{\pm}) = 0$

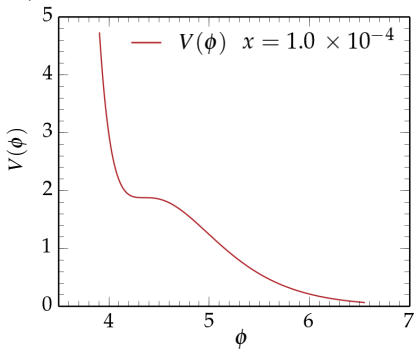
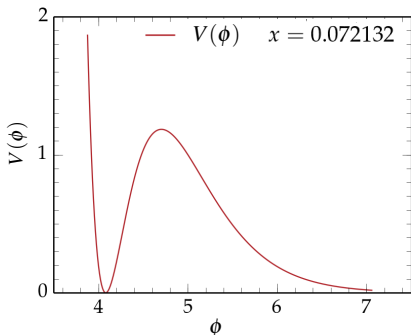
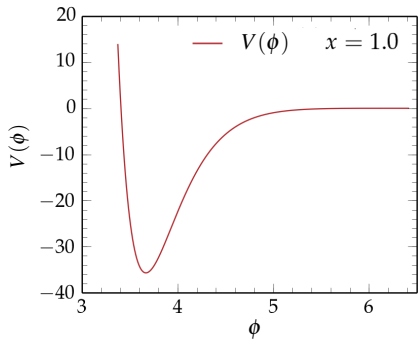
$$\phi_+ - \phi_- = \sqrt{2/3} (W_0(-e^{-x-1}) - W_{-1}(-e^{-x-1}))$$

$W_{0/-1}$: Lambert functions satisfying $W(xe^x) = x$

$$\frac{V(\phi_+)}{V(\phi_-)} = \frac{(W_0(-e^{-x-1}))^3 (2+3W_{-1}(-e^{-x-1}))}{(W_{-1}(-e^{-x-1}))^3 (2+3W_0(-e^{-x-1}))}$$

- slow roll parameter $\eta(\phi_{-/+}) = \frac{V''(\phi_{-/+})}{V(\phi_{-/+})} = -9 \frac{1+W_{0/-1}(-e^{-x-1})}{\frac{2}{3}+W_{0/-1}(-e^{-x-1})}$ [25]

successful inflation possible around the minimum from the inflection point



[26] [27]

Inflation possibilities

- Friedmann equations with time replaced by the inflaton \Rightarrow

Hubble parameter $\rightarrow H'(\phi) = \mp \frac{1}{\sqrt{2}} \sqrt{3H^2(\phi) - V(\phi)}$

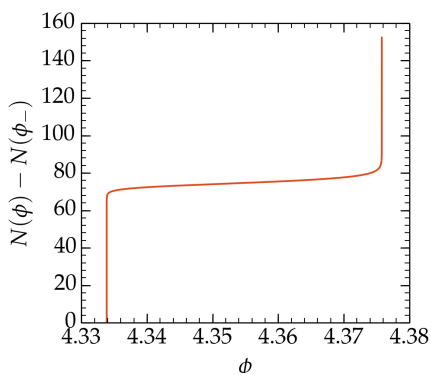
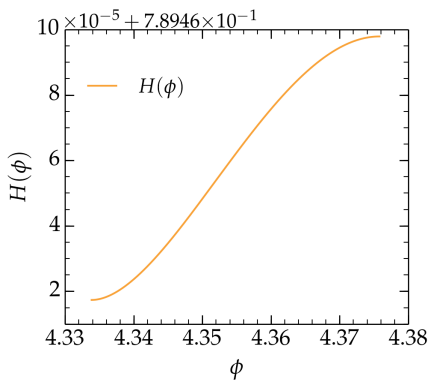
- slow-roll parameters: $\eta(\phi) = \frac{V''(\phi)}{V(\phi)}$, $\epsilon(\phi) = \frac{1}{2} \left(\frac{V'(\phi)}{V(\phi)} \right)^2$

- number of e-folds by the end of inflation: $N(\phi) = \int_{\phi_{end}}^{\phi} \frac{d\phi}{\sqrt{2\epsilon}}$

Observational constraints at the horizon exit $\phi = \phi_*$:

- 1 $N_* \simeq 50 - 60$
- 2 spectral index of power spectrum $n_S - 1 = 2\eta_* - 6\epsilon_* \simeq -0.04$
- 3 amplitude of scalar perturbations $\mathcal{A}_S = \frac{V_*}{24\pi^2 \epsilon_*} \simeq 2.2 \times 10^{-9}$

\Rightarrow inflation possible around the minimum from the inflection point [20]



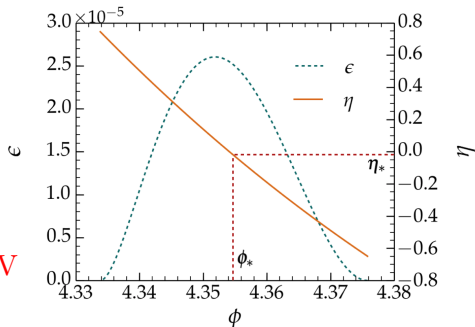
$$x = 3.3 \times 10^{-4}; \quad \eta(\phi_*) = -0.02$$

ϕ_* near the inflection point

$\Delta\phi \simeq 0.02$: small field

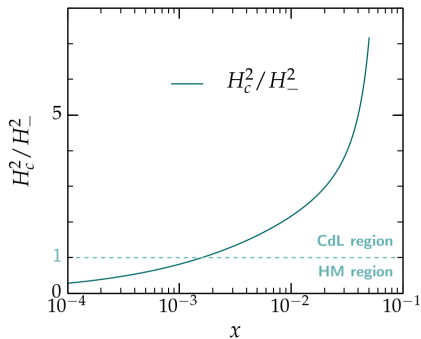
$$\Rightarrow r \simeq 4 \times 10^{-4}$$

$$H_* \simeq 5 \times 10^{12} \text{ GeV}$$



dS vacuum metastability [23]

- through tunnelling $H_c > H_-$ Coleman - de Luccia instanton
- over the barrier $H_c < H_-$ Hawking - Moss transition



$$\frac{H_c^2}{H_-^2} \equiv -\frac{3V''(\phi_+)}{4V(\phi_-)}$$

$$\text{HM region: } \Gamma \sim e^{-B}; \quad B \simeq \frac{24\pi^2}{V} \frac{\Delta V}{V}$$

$$\frac{\Delta V}{V} \simeq 24\sqrt{2}x^{3/2} \Rightarrow$$

$$B \simeq 3 \times 10^9 \text{ for } x \simeq 3 \times 10^{-4}$$

End of inflation with waterfall field

Hybrid scenario

$$V(\phi, S) = V(\phi) + \frac{1}{2}m_S^2(\phi)S^2 + \frac{\lambda}{4}S^4$$

$$\phi > \phi_c : m_S^2 > 0 \Rightarrow \langle S \rangle = 0, \quad V(\phi, 0) = V(\phi)$$

$$\phi < \phi_c : m_S^2 < 0 \Rightarrow \langle S \rangle = \pm \frac{|m_S|}{\sqrt{\lambda}}, \quad V(\phi, \langle S \rangle) = V(\phi) - \frac{m_S^4(\phi)}{4\lambda}$$

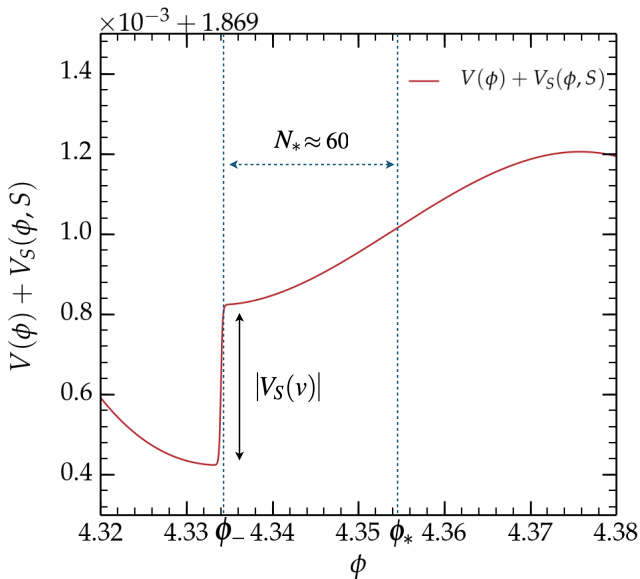
ϕ_c : near the minimum of $V(\phi)$

waterfall field S : open string state on D7-branes

negative contribution to m_S^2 : from internal magnetic fluxes
along the world-volumes [32]

I.A.-Lacombe-Leontaris '21

End of inflation with waterfall field



Example: $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ with 3 sets of $D7$ -branes

	$(45)_1$	$(67)_2$	$(89)_3$		$(45)_1$	$(67)_2$	$(89)_3$	
$D7_1$	\cdot	\times	\times	\rightarrow	$D7_1$	\cdot	\otimes	\times
$D7_2$	\times	\cdot	\times		$D7_2$	\times	\cdot	\otimes
$D7_3$	\times	\times	\cdot		$D7_3$	\otimes	\times	\cdot

Turn on magnetic fields: $H_a^{(i)}$ on the stack $D7_a$ along the i -th torus T_i^2

Dirac quantisation condition:

$$m_a^{(i)} \int H_a^{(i)} = 2\pi n_a^{(i)} \Rightarrow H_a^{(i)} = 2\pi k_a^{(i)} / \mathcal{A}_i \quad ; \quad k_a^{(i)} = n_a^{(i)} / m_a^{(i)} \in \mathbb{Q}$$

Frequency shift of charged oscillator modes:

$$\zeta_a^{(i)} = \frac{1}{\pi} \text{Arctan}(2\pi\alpha' q_a H_a^{(i)}) \sim q_a \frac{k_a^{(i)}}{\mathcal{A}_i} \quad (\text{large area limit})$$

$q_a = \pm 1, 0$: $U(1)$ charges of open string endpoints

Masses of charged lower lying states

- same stack double charged (between brane and its orientifold image)

$$D7_a^{(i)}-D7_a^{(i)} : m^2 = -2|\zeta_a^{(i)}|$$

- brane intersections $D7_a^{(i)}-D7_b^{(j)} : m^2 = \pm (|\zeta_a^{(i)}| - |\zeta_b^{(j)}|)$

Tachyon elimination \Rightarrow

- brane intersections: equality of magnetic fields $|\zeta_a^{(i)}|$

- same stack: turn-on (discrete) Wilson lines $A_a^{(i)}$; $|A_a^{(i)}|^2 = \alpha_a^2/\mathcal{A}_i$

$$\text{and brane separations } x_a^{(i)} ; |x_a^{(i)}|^2 = y_a^2 \mathcal{A}_i$$

$$\Rightarrow m^2 = -2|\zeta_a^{(i)}| + |A_a^{(j)}|^2 + |x_a^{(k)}|^2$$

Appearance of tachyons decreasing the volume \Rightarrow waterfall fields as open string states

	(45) ₁	(67) ₂	(89) ₃		(45) ₁	(67) ₂	(89) ₃	
$D7_1$	\cdot	\otimes	\times	\longrightarrow	$D7_1$	\cdot	\otimes	$\times_{A_1^{(3)}}$
$D7_2$	\times	\cdot	\otimes		$D7_2$	\times	$\cdot_{x_2^{(2)}}$	\otimes
$D7_3$	\otimes	\times	\cdot		$D7_3$	\otimes	$\times_{A_3^{(2)}}$	\cdot

$\mathcal{A}_i \equiv r_i \mathcal{V}^{1/3}$ with $r_1 r_2 r_3 = 1 \Rightarrow$ (large volume)

$$m_{11}^2 \approx \left(-\frac{2|k_1^{(2)}|}{\pi r_2} + \frac{\alpha_1^2}{r_3} \right) \mathcal{V}^{-1/3} \quad ; \quad m_{33}^2 \approx \left(-\frac{2|k_3^{(1)}|}{\pi r_1} + \frac{\alpha_3^2}{r_2} \right) \mathcal{V}^{-1/3}$$

$$m_{22}^2 \approx -\frac{2|k_2^{(3)}|}{\pi r_3 \mathcal{V}^{1/3}} + y_2^2 r_2 \mathcal{V}^{1/3}$$

α_1, α_3 can be arranged to make positive m_{11}^2, m_{33}^2 for all \mathcal{V}

However m_{22}^2 becomes tachyonic decreasing $\mathcal{V} \Rightarrow$ waterfall field

Conclusions

New mechanism of moduli stabilisation is string theory (type IIB)

- perturbative: weak coupling, large volume
- based on log corrections in the transverse volume of 7-branes due to local tadpoles induced by localised gravity kinetic terms arising only in 4 dimensions!
- can lead to de Sitter vacua in string theory
explicit counter-example to dS swampland conjecture
- inflation possible around the minimum from the inflection point
- realisation of hybrid inflation to lower the vacuum energy